$\nabla C = i(2(2)+1+2(1)) + j(2(1)) + k(1+(2)(1))$

$= i(4+1+2) + j(2) + k(1+2)$

$= 7i + 2j + 3k$

v) curl curl A

curl $A = \nabla \times A$

$$= \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & (xy+yz) & xz^2 \end{vmatrix}$$

$= i(0-y) - j(z^2-0) + k(y-x^2)$

$= -yi - z^2 j + k(y-x^2)$

At (1, 2, 1)

curl $A = -2i - j + k$

curl curl $A = \nabla \times (\nabla \times A)$

$\nabla \times (\nabla \times A) = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y & -z^2 & (y-x^2) \end{vmatrix}$

$= i(1+2z) - j(-2x-0) + k(0+1)$

$= i(1+2z) + 2xj + k$

At points (1, 2, 1)

$\nabla \times (\nabla \times A) = i(1+2(1)) + 2(1)j + k$

$= 3i + 2j + k$

$A + (1, 2, 1)$

$\frac{\partial \phi}{\partial x} = 6(1)(2) + (2)(1) = 12 + 2 = 14$

$\frac{\partial \phi}{\partial y} = 3(1)^2 + (1)(1) - 8(2)(1)^2 = 3 + 1 - 16 = -12$

$\frac{\partial \phi}{\partial z} = (1)(2) - 8(2)^2(1) = 2 - 32 = -30$

$\nabla \phi = 14\hat{i} - 12\hat{j} - 30\hat{k}$

ii) $\nabla \cdot A = \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}$

$A = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

$\nabla \cdot A = 2xy + (x + z) + 2xz$

$A + (1, 2, 1)$

$\nabla \cdot A = 2(1)(2) + (1 + 1) + 2(1)(1)$

$= 4 + 2 + 2 = 8$

iii) $\nabla \times B$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & -3xz & 2xy \end{vmatrix}$$

$= \hat{i}(2x + 3x) - \hat{j}(2y - y) + \hat{k}(-3z - z)$

$= 5x\hat{i} - y\hat{j} - 4z\hat{k}$

$A + (1, 2, 1)$

$\nabla \times B = 5\hat{i} - 2\hat{j} - 4\hat{k}$

iv) grad div A

grad $(2xy + (x + z) + 2xz)$

let div $A = C = \nabla A$

$\nabla(\nabla A) = \nabla C = \hat{i}\frac{\partial C}{\partial x} + \hat{j}\frac{\partial C}{\partial y} + \hat{k}\frac{\partial C}{\partial z}$

$= \hat{i}(2y + 1 + 2z) + \hat{j}(2x) + \hat{k}(1 + 2x)$

$A + (1, 2, 1)$

Name : Ukuku David Ebi
Matric No : 16/ENG01/022
Dept : Chemical
Course Code : ENG 282

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Assignment III

1 A mathematical model is a description of a system using mathematical concepts and language. Therefore modelling is the process of setting up a model, solving it mathematically and interpreting the result in physical and other terms

b i) Exponential growth / decay (ex of DDE)
b ii) Mixing problems

2 $r = (t^2 + 3t)i - 2\sin 3t\,j + 3e^{2t}k$

i) $\frac{dr}{dt} = (2t + 3)\hat{i} - 6\cos 3t\,\hat{j} + 6e^{2t}\hat{k}$

ii) $\frac{d^2r}{dt^2} = 2\hat{i} + 18\sin 3t\,\hat{j} + 12e^{2t}\hat{k}$

iii) $\left.\frac{d^2r}{dt^2}\right|_{t=0} = 2\hat{i} + 12\hat{k}$

$\left|\frac{d^2r}{dt^2}\right| = \sqrt{2^2 + 12^2} = \sqrt{4 + 144} = \sqrt{148} = 2\sqrt{37} = 12.17$

3 $A = x^2y\,i + (xy + yz)\,j + xz^2k$
$B = yz\,i - 3xz\,j + 2xy\,k$
$\phi = 3x^2y + xyz - 4y^2z^2 - 3$

i) $\nabla\phi = \frac{\partial\phi}{\partial x}i + \frac{\partial\phi}{\partial y}j + \frac{\partial\phi}{\partial z}k$

$\frac{\partial\phi}{\partial x} = 6xy + yz \qquad \frac{\partial\phi}{\partial y} = 3x^2 + xz - 8yz^2$

$\frac{\partial\phi}{\partial z} = xy - 8y^2z$